**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

**Ans: here *μ* = 45, *σ* = 8 . The probability that the time required for the servicing transmission for car will be ready within 1 hour is p(c ) = 0.7340**

**And the probability car will not be ready is 1- p(c ) = 0.2676**

**Option B is correct**

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.

**Ans: False. If there are more employees older than 44 then probability for that is greater than probability of employees of age in 38 to 44. Here P(age>44) =0.1586 < P(38<age<44) =0.3413**

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

**Ans: True.**

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

**Ans: Here, *X1* ~ *N*(μ, σ2) then the distribution of 2 *X*1 is *N*(2μ, 4σ2) and if *X*2 ~ *N*(μ, σ2) then *X*1 + *X*2  has the distribution *N*(2μ, 2σ2). We have to find the distribution of 2 X1-(X1+X2),**

**2 X1-(X1+X2) ~ *N*(0, 2σ2)**

**Here parameters are mean = 0, variance=2σ2**

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

**Ans: Since we need to find out the values of a and b, which are symmetric about the mean, such that the probability of random variable taking a value between them is 0.99, we have to work out in reverse order. The Probability of getting value between a and b should be 0.99. So the Probability of going wrong, or the Probability outside the a and b area is 0.01 (ie. 1-0.99). The Probability towards left from a = -0.005 (ie. 0.01/2). The Probability towards right from b = +0.005 (ie. 0.01/2). So since we have the probabilities of a and b, we need to calculate X, the random variable at a and b which has got these probabilities. By finding the Standard Normal Variable Z (Z Value), we can calculate the X values. Z=(X- μ) / σ For Probability 0.005 the Z Value is -2.57 (from Z Table). Z \* σ + μ = X**

**Z(-0.005)\*20+100 = -(-2.57)\*20+100 = 151.4**

**Z(+0.005)\*20+100 = (-2.57)\*20+100 = 48.6**

**Hence option D is correct.**

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

**Ans: If Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) then Profit1+Profit2 ~ N(12,52).**

**Here mean profit in Rs is 540 and standard deviation is 225**

**Then the range such that it contains 95% probability for the annual profit of the company is (99.0081, 980.9918965)**

1. **Specify the 5th percentile of profit (in Rupees) for the company**

**Ans: *To compute 5th Percentile, we use the formula X=μ + Zσ;***

***5 percentile= -1.645 from z table,* X= 540+(-1.645\*(225) = 170**

**The 5th percentile value is 170**

1. Which of the two divisions has a larger probability of making a loss in a given year?

**Ans: Probability of Division 1 making a loss i.e P(X<0) is 0.047790**

**Probability of Division 1 making a loss i.e P(X<0) is 0.040059**